More Programming with FORTRAN

I. Objective

To write a FORTRAN program to solve a set of coupled differential equations

II. Introduction

Today you will write a FORTRAN program to solve a set of coupled differential equations. Today's project is related to chaos theory, in that the solution of the equations is a **strange attractor.** The graphs you will make are very famous, and were first published by E. Lorentz[1] . Lorentz was modeling the dynamics of the atmosphere, and came to the conclusion that the sensitivity to initial conditions of the solution made unlikely the possibility of long term weather prediction.

Our goal will be to write a program to solve the equations and then study the solution as the initial conditions are changed.

III. Exercises

A. Playing with Chaos

Objective:	to write a FORTRAN program that solves the systems of differential equations below
What to do:	follow the instructions below
What to turn in to your instructor:	your log book, a copy of the program **lor.f**, and graphs as described below
What to put in log book:	problems, solutions, interesting facts, observations on the Lorentz attractor

(1) Coupled Differential Equations: In his 1963 paper, E. Lorentz solved the following system of coupled linear differential equations:

$$\frac{dx}{dt} = A(y - x)$$

$$\frac{dy}{dt} = Bx - y - xz$$

$$\frac{dz}{dt} = xy - Cz$$

[1] E. Lorentz, *Journal of Atmospheric Sciences,* **20**, 130 (1963)

where A=10, B=28, and C=8/3. These equations arise from a model of convection currents in the atmosphere. In these equations x is proportional to the convection current, y is proportional to the temperature difference between the ascending and descending currents, and z is proportional to the distortion of the vertical temperature profile from linearity.

(2) Copy Files from the Physics 232 Locker: Your program will read several parameters from an input file called **in2**. Copy the file **in2** to your working directory from the locker Physics 232 locker **/home/physics/phys232.**

(3) FORTRAN Program: Write a FORTRAN program called **lor.f** that reads the parameters from the input file **in2** and solves the above coupled set of differential equations using the Euler Method (see Appendix 1). Your program should include the following:

- A main program
- A subroutine called **euler** for performing the Euler's Method calculations
- A set of three function subroutines **xfunc, yfunc,** and **zfunc** that are called by **euler** and calculate the necessary functions of the Euler Method (see Appendix 1).
- A set of **write** statements that write the data pair **x,y** to file **fort.7**, the data pair **y,z** to file **fort.8**, and the data pair **x,z** to **fort.9**.

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You may use the FORTRAN code from last week's assignment as a guide.

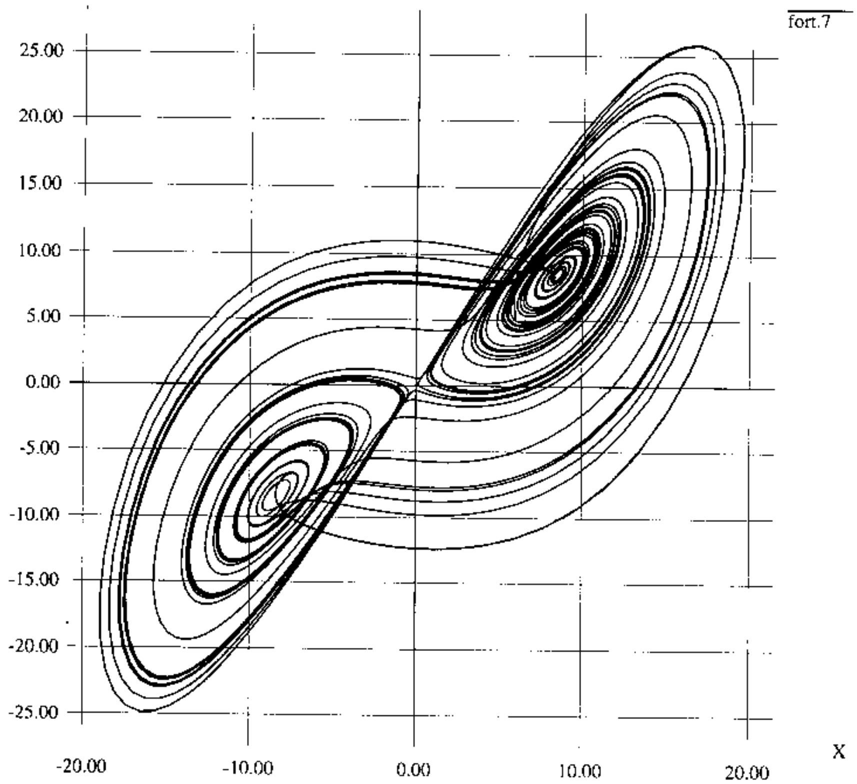
(4) Compiling and Running the Program: Compile the program as explained in last week's handout and create an executable file called **lor**.

The programs we ran last week completed their runs within a few seconds. Today's program may take several minutes to run, depending on the values you give it. For this reason we will run the program in the **background**, thus freeing the workstation for other tasks during program execution. The command for this is:

lor<*in2*&

Command	What it does
lor<in2	executes the program **lor** which reads the file **in2**
&	indicates the program is to be run in the background

(5) Plotting the Results: For the parameters given in **in2**, use **xgraph** to plot each of the three data sets the program created. You should find that the plot of y versus x looks something like the figure below. Submit this plot to your instructor.



Try plotting z versus y and z versus x as well.

(6) Exploring the Strange Attractor: The above plot of y versus x reveals a **strange attractor**. The term **attractor** refers to the asymptotic limit of the solution as time goes to infinity. Strange attractors exhibit great sensitivity to initial conditions[2].

Study the sensitivity to initial conditions by observing at leaste three plots of y versus x for different values of *x(0), y(0),* and *z(0).* Draw sketches in your log book indicating roughly what the asymptotic behavior looks like for the different initial conditions you chose.

In his paper, Lorentz stated that for the values of A, B, and C given in **in2** the "points of steady convection ... are $\left(6\sqrt{2},\ 6\sqrt{2},\ 27\right)$ and $\left(-6\sqrt{2},\ -6\sqrt{2},\ 27\right)$..."

What will happen if you give either of the above points as an initial condition? Describe the result in your log book.

Play around with the parameters A, B, and C in the input file and observe the effects on the attractor. Be careful! Your solutions may blow up for certain values.

[2] E. Rietman, *Exploring the Geometry of Nature,* Chapter 2.

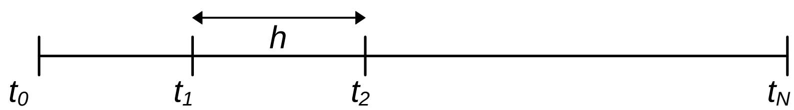
Describe your observations in your log book and submit to your instructor a graph of your favorite attractor.

IV. Appendix 1: The Euler Method

Coupled equations of the form

$$\frac{dx}{dt} = X(x,y,z)$$
$$\frac{dy}{dt} = Y(x,y,z)$$
$$\frac{dz}{dt} = Z(x,y,z)$$

may be numerically solved for *x(t), y(t),* and *z(t)* using the **Euler Method**[3] if the initial conditions *x(0), y(0),* and *z(0))* are known. The t axis is divided into N intervals of equal length $h = \frac{t_N - t_0}{N}$ as shown below. This method may be summarized by the recursion



relations in the table below. We use the notation $x_i \equiv x(t_i)$, and similar notation for y and z.

Euler Method Recursion Relations for *x, y,* and *z*
$x_0 \equiv x(0)$ $x_{i+1} = x_i + hX(x_i, y_i, z_i)$
$y_0 \equiv y(0)$ $y_{i+1} = y_i + hY(x_i, y_i, z_i)$
$z_0 \equiv z(0)$ $z_{i+1} = z_i + hZ(x_i, y_i, z_i)$

[3] See S. E. Koonin, *Computational Physics,* Chapter 2, for a complete description.